An alternative to the Generalized Second Price Auction

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Motivation

- The generalized second price auction (GSP) is *not* incentive compatible
  - Often result in an inefficient outcome when CTRs of ad-slots are similar
  - No evidence of efficiency was found in empirical studies (Börgers et al., 2013)
- A slight modification may improve the efficiency.

GSP auction – keyword auction

- keyword auctions allocate ad-slots.
- They take place continuously in real time
- Nash Equilibrium under complete information is used to analyze these auctions.

The generalized second price auction

Ex) 2 ad-slots and 3 bidders.

click through rates (CTRs) of the two positions. $(c_1, c_2) = (200,150)$

Value per click of the three bidders $(v_1, v_2, v_3) = (10, 5, 2)$

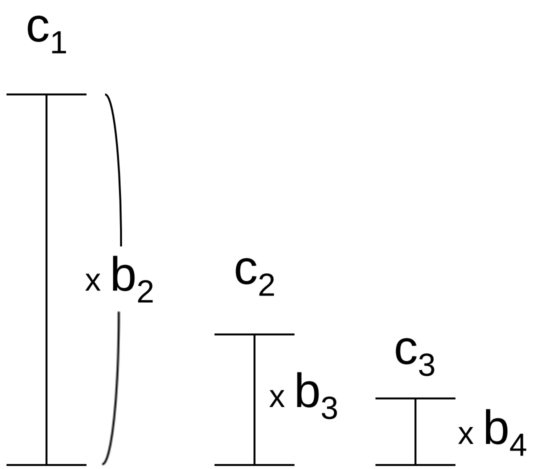
Click-through rates (CTRs)	Bids	Payment per-click	Payoff
200	b_1 (10)	5	(10 – 5) x 200 = 1000
150	b_2 (5)	2	(5-2) x 150 = 450
-	b_3 (2)	-	

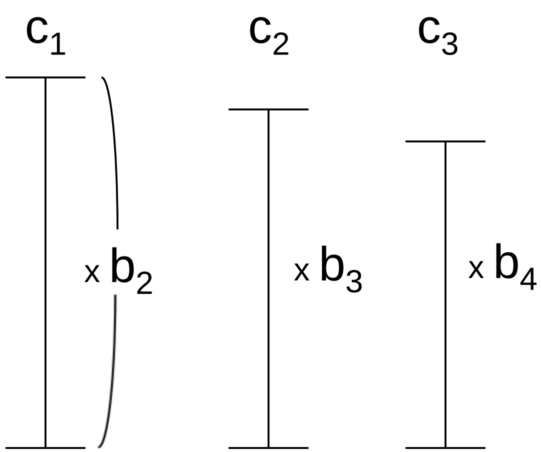
- Truthful-bidding is not a dominant strategy.
- If bidder 1 submits 3, he gets the second position and his payoff becomes (10-2)*150 = 1200 (>1000)

Strong incentive to bid low -> result in inefficient outcomes

The generalized second price auction

- Incentive to bid low becomes serious when CTRs are similar

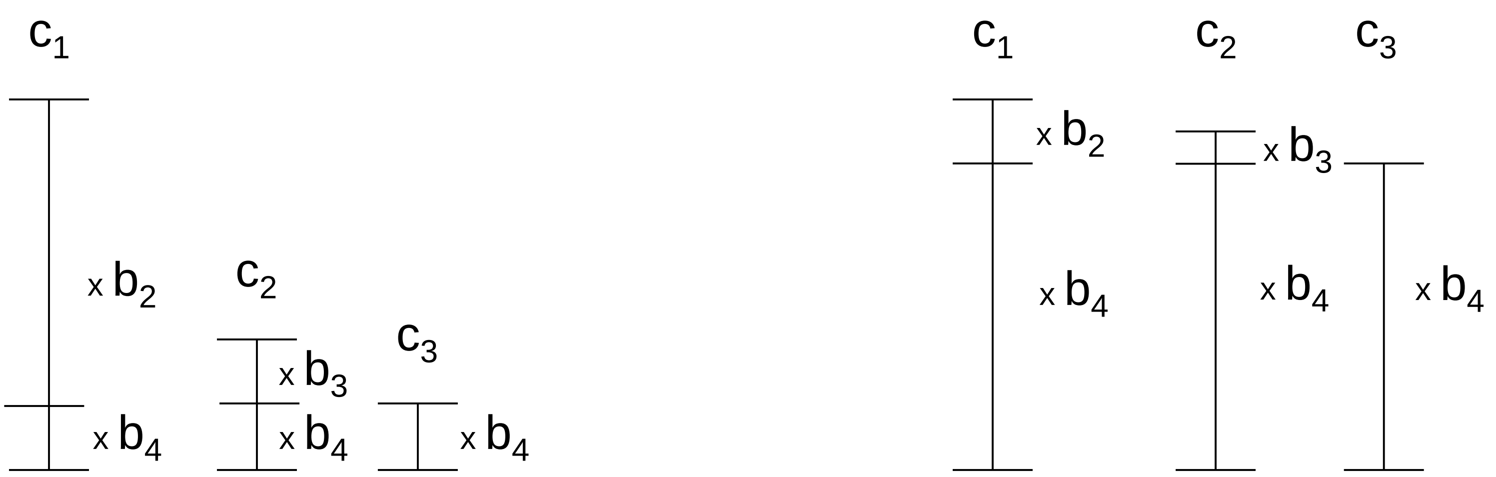




Any better option? (other than VCG)

An alternative

- A possible alternative



- For the base CTRs, set the same price
- For additional CTRs, GSP rule applies

-> Is this better? In what sense?

Compare the two auction rules

- Both of them do not have dominant strategy.
- I will assume that an auction rule where value bidding is more likely to be a NE is better.
  - When value bidding is a NE, outcomes are more efficient (Che et al. 2017)

Compare the two auction rules

- Number of ad-slots : *J*
- Number of bidders : *I*
- Let $\gamma = c_{j+1}/c_j$ for all *j = 1,2,…,J-1*
  - This is a usual assumption (Edelman & Ostrovsky, 2007)

- GSP auction

  When $\gamma \to 0$: value bidding is always a NE

  $\gamma \to 1$: value bidding is never a NE.

The generalized second price auction

Ex) J=2

Value bidding is a NE *iff*

$$(v_1 - v_2)\, c_1 \geq (v_1 - v_3)\, c_2$$

$$\frac{v_1 - v_2}{v_1 - v_3} \geq \frac{c_2}{c_1} \geq \gamma$$

$$pr\left(\frac{v_1 - v_2}{v_1 - v_3} \geq \gamma\right) \rightarrow 1, \qquad when\, \gamma \rightarrow 0$$

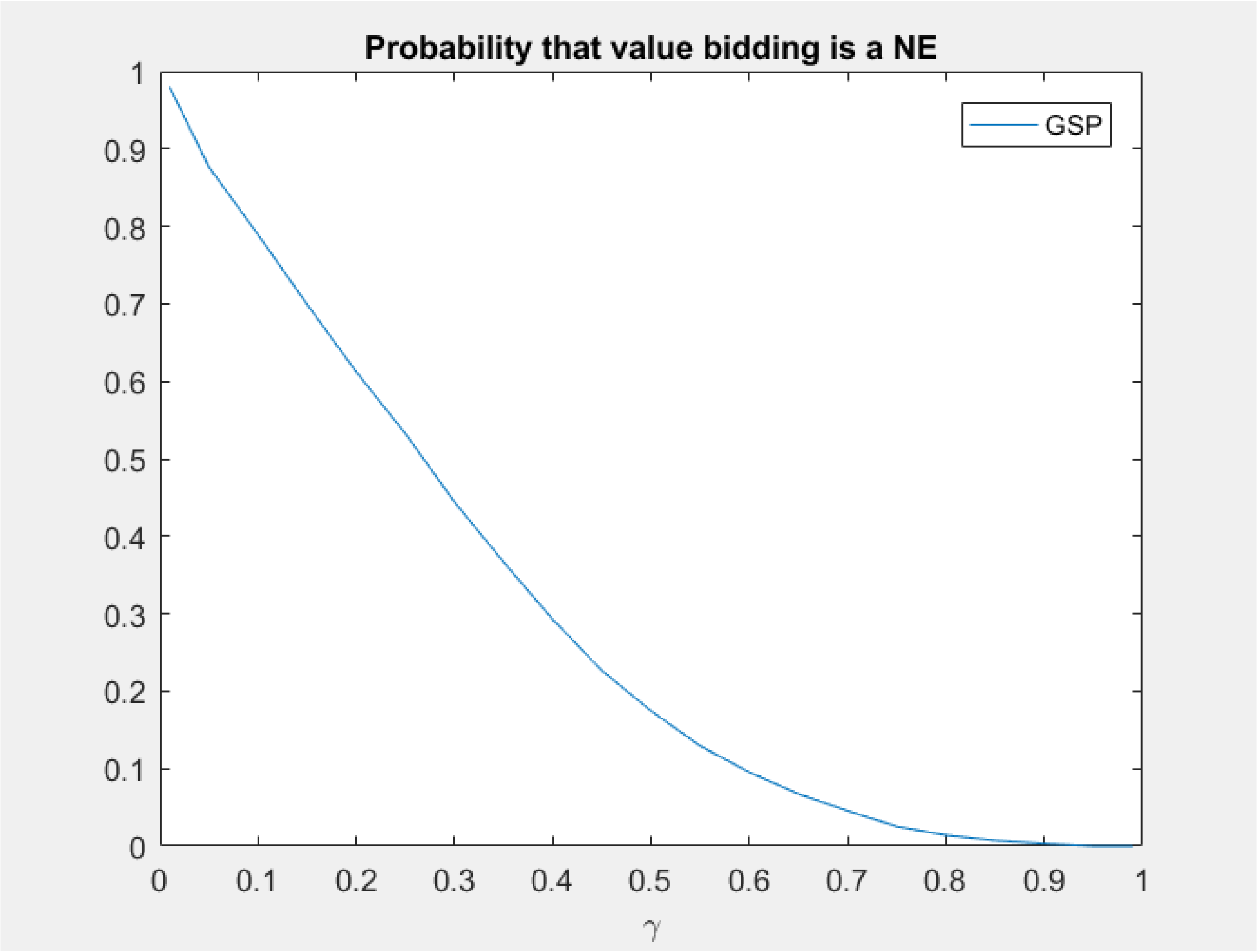
$$pr\left(\frac{v_1 - v_2}{v_1 - v_3} \geq \gamma\right) \rightarrow 0, \qquad when\, \gamma \rightarrow 1$$

The generalized second price auction

$J = 3$

$I = 4$

$v_i \sim U[0,100]$



The alternative

- Number of ad-slots : *J*
- Let $\gamma = c_{j+1}/c_j$ for all *j = 1,2,…,J-1*
  - This is a usual assumption (Edelman & Ostrovsky, 2007)

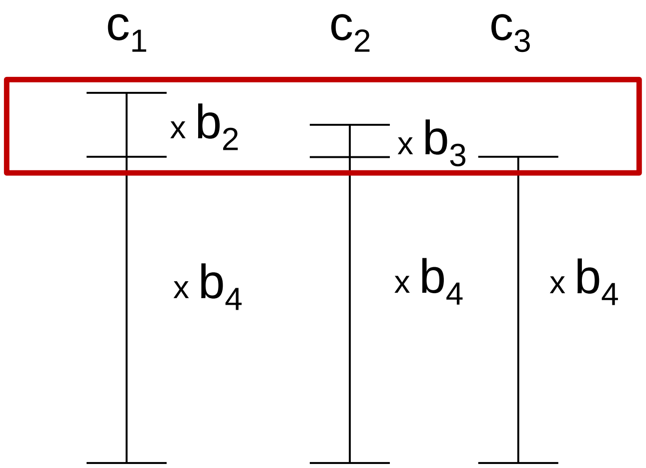
- The alternative

  When $\gamma \to 0$: value bidding is always a NE.

  $\gamma \to 1$: value bidding is always an ϵ equilibrium. (for any fixed ϵ)

The alternative

- Why not a NE but an ϵ −equilibrium?



When $\gamma \to 1$, additional CTRs becomes similar, so incentive bid low in GSP still survives.

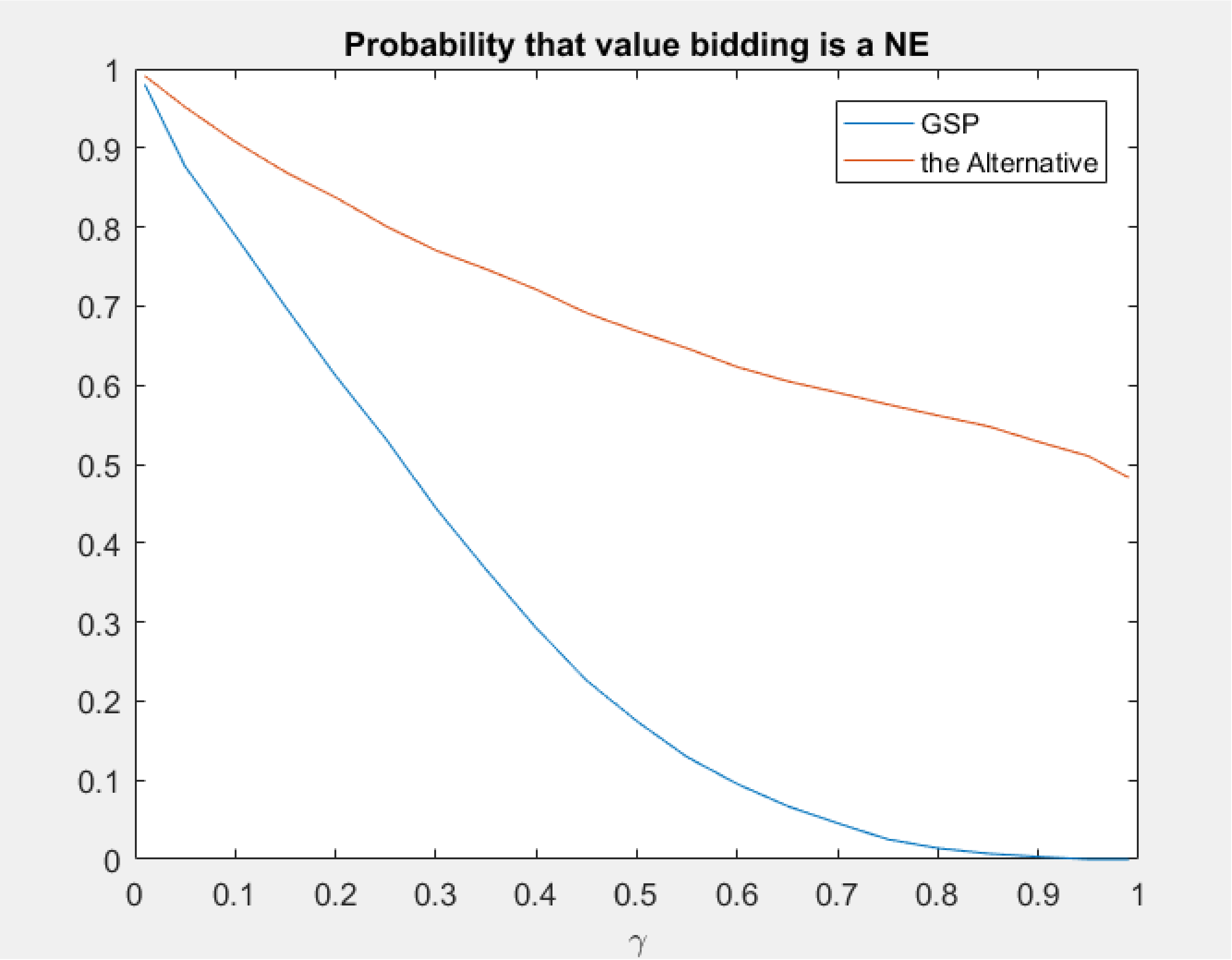
But the gain from deviation is negligible

Compare the two auction rules

J= 3

I= 4

v_i ~ U[0,100]



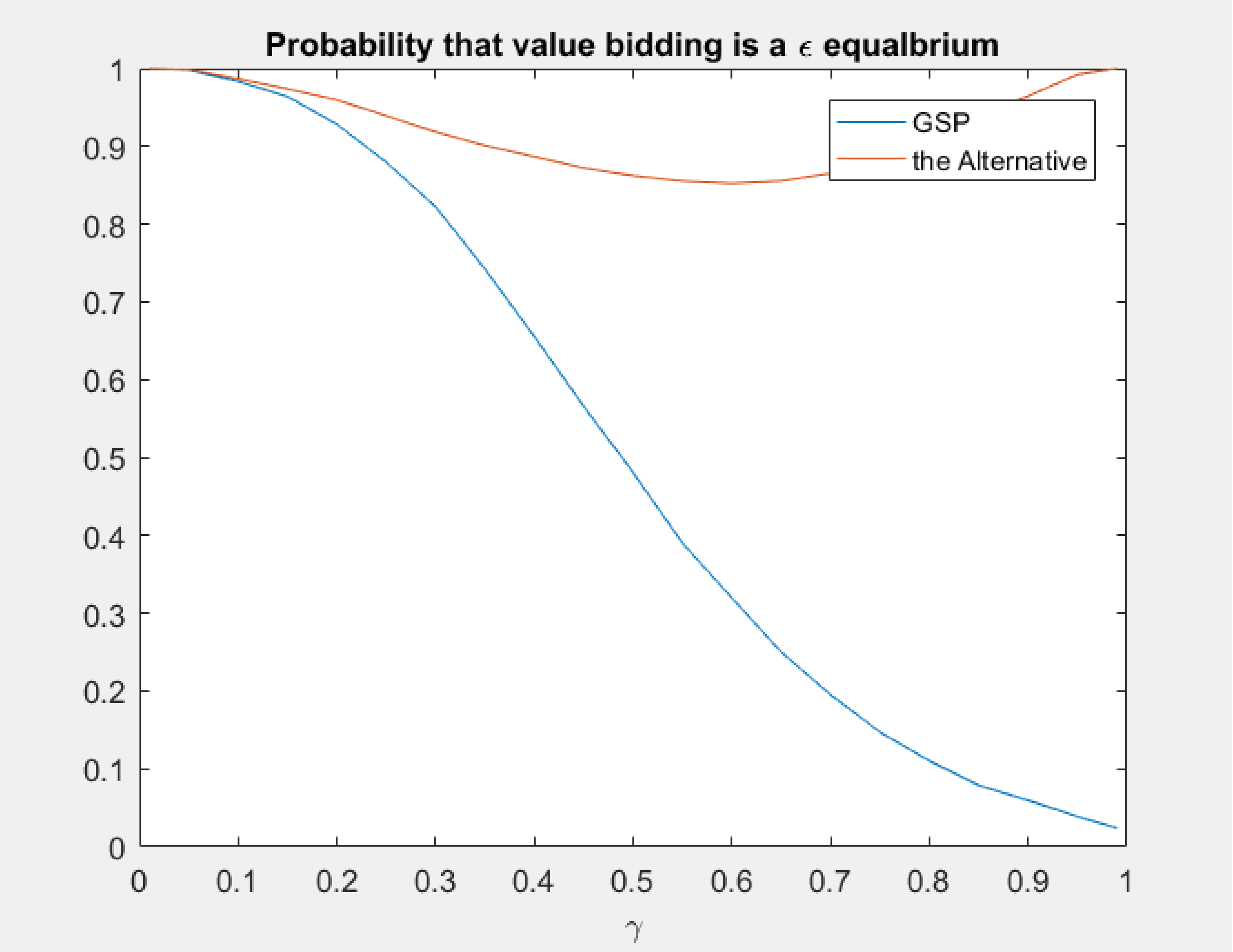
Compare the two auction rules

J= 3

I= 4

v_i ~ U[0,100]

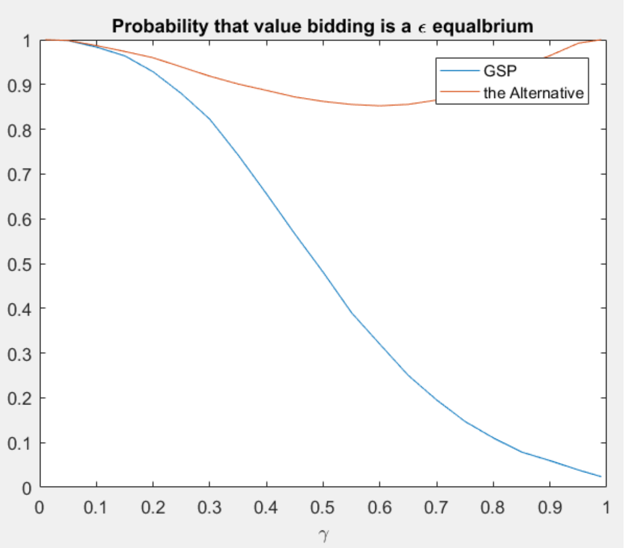
ϵ = a small number

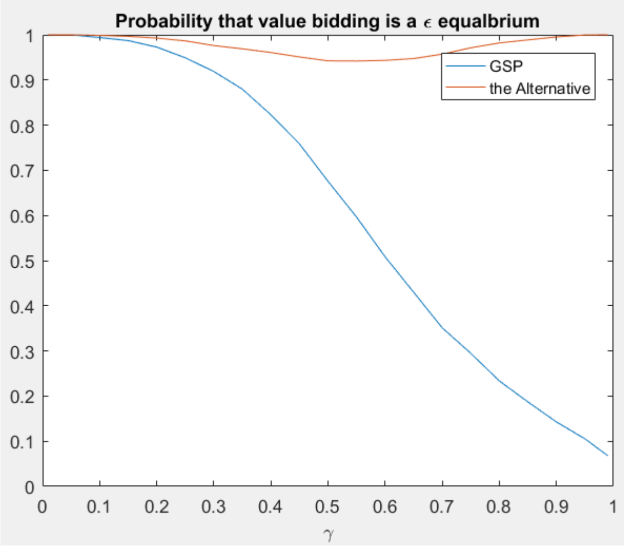


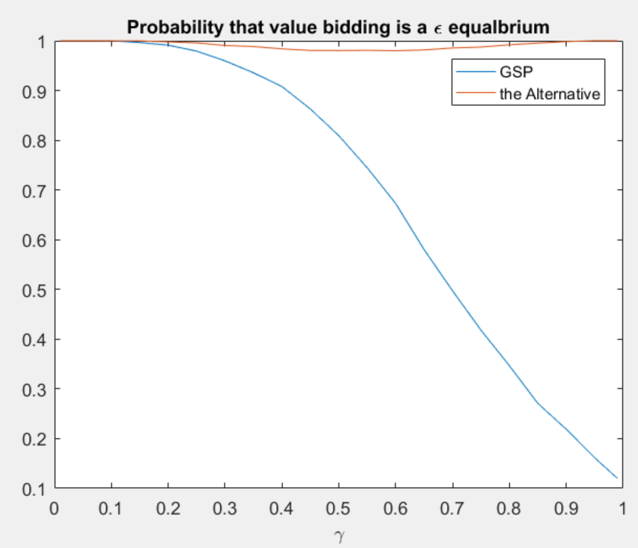
Compare the two auction rules

J=3, I=4 *J=3, I=8* *J=3, I=12*

ϵ = a small number







Summary / Dissusion

- The alternative auction can be better, especially when CTRs are similar to each other
- Tells not much about when γ is intermediate

- More simple and better auction rule?
- Related theory concept?
- Revenue comparison?
- Experiment?
- Anything else?

Another Summary

GSP	M-GSP	VCG
Bidding above value is dominated	-	Value bidding is a dominant strategy
Equivalent to VCG when n=1	Equivalent to VCG when n=2	
When γ->0, Value bidding is NE	When γ->0, Value bidding is NE When γ->1, Value bidding is NE	

Is it possible to show that pr (GSP) > pr (alternative) ?
When more excessive bidders -> infinity, the alternative will be strategy proof ?
equilibrium characterization?
Value bidding is a NE vs strategy proofness?

cf. Vickrey auction

- Vickrey auction